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An Invariant Counting the Number of Critical Points

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A non-negative integer invariant γ estimating from below the number of geometrically different critical points of a smooth function $f : \mathbb{B}^2 \to \mathbb{R}$ is defined. It depends on combined $C^0 + C^1$ type conditions on the boundary $\partial(\mathbb{B}^2) = \mathbb{S}^1$, that we call *ribbons* here. It turns out to be an alternative to the degree of the gradient map and almost independent from it. Note that the degree does not guarantee multiple critical points, unlike the ribbon invariant γ. In fact, our invariant is counting the number of essential components of the critical set, rather than simply the number of critical points. Various estimates of γ are established. Some other *ribbon type* invariants of geometrical nature are defined and investigated. All these invariants turn out to be more combinatorial, rather than algebraic, in nature. A parallel algorithm for the calculation of the ribbon invariants, relying on ribbon's discretization, is proposed. Interconnections with some different areas, such as the theory of immersed curves in the plane, Arnold's $J^{\pm}$ invariants or independent domination in graphs, are commented. Note finally that our invariant γ is quite different in nature from the Ljusternik-Schnirelmann category or the Nielsen number, although the latter have a similar task - the estimation of the critical set, or the fixed points set, respectively.