Scattered Spaces and Selections

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If the Vietoris hyperspace $\mathcal{F}(X)$ of the nonempty closed subsets of a regular scattered space X admits a continuous mapping $f : \mathcal{F}(X) \to X$ selecting an isolated point of each member of $\mathcal{F}(X)$ (i.e. a *continuous zero-selection*), then so does $\mathcal{F}(Z)$ for every nonempty $Z \subset X$. The present talk aims to discuss the inverse problem by showing that X is a scattered space provided $\mathcal{F}(Z)$ has a continuous selection for every nonempty countable $Z \subset X$. The essential result behind this is that a crowded regular space X contains a copy of the rational numbers provided its Vietoris hyperspace $\mathcal{F}(X)$ has a continuous selection. Some related problems and applications will be discussed as well.