

On a Theorem of Prodanov

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In 1971 Prodanov characterized the compact groups of p -adic integers as the only infinite compact abelian groups such that all their subgroups are minimal. This raised the problem on whether one can appropriately remove the restraint on commutativity and replace compactness by the weaker assumption of local compactness. We discuss this problem, providing various solutions using the notion of locally minimal group, introduced by Morris and Pestov in 1997.

In particular, we extend Prodanov’s theorem in two directions:

(a) we show that a locally compact abelian group such that all subgroups of L are locally minimal is either a Lie group or has an open subgroup isomorphic to the compact groups of p -adic integers for some prime p ;

(b) we characterize the connected Lie groups as the only connected locally compact groups such that all their subgroups are locally minimal.

Other characterizations of Lie groups (based on the control of zero-dimensional subgroups) will be discussed as well.