

## A Relation Between Exposed Points and Extremal Points of Convex Sets

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Let  $V$  be a Euclidean space or the Hilbert space consisted of square summable sequences. Let  $k$  be a natural number with  $k < \dim V$ , let  $B$  be convex and closed in  $V$  and let  $P$  be a collection of linear  $k$ -subspaces of  $V$ . A set  $C$  in  $V$  is called a *P-imitation* of  $B$  if  $B$  and  $C$  have identical orthogonal projections along every  $p$  in  $P$ . An *extremal* point of  $B$  with respect to the projections under  $P$  is a point that all closed subsets of  $B$  that are  $P$ -imitations of  $B$  have in common. A point  $x$  of  $B$  is called *exposed* by  $P$  if there is a  $p$  in  $P$  such that  $x + p$  and  $B$  have in common only  $x$ .

The main result we present is that all extremal points are limits of sequences of exposed points whenever  $P$  is open.

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