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Two-component NLS equations: Hamiltonian Properties and Generalized Fourier Transforms

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Recently several new 2-component integrable NLS equations (2-NLSE) have been discovered [1, 2]. Our main result is to demonstrate a family of 2-NLSE related to the symmetric spaces $SO(2r+1)/SO(2r-3)\otimes SO(4)$ of **BD.I**-type. The relevant Lax operators are of the form:

$$L\psi \equiv i\frac{\partial\psi}{\partial x} + (Q(x,t) - \lambda J)\psi(x,t,\lambda) = 0, \quad Q(x,t) = \begin{pmatrix} 0 & \vec{q}^T & 0 \\ -\vec{p} & 0 & s_0\vec{q} \\ 0 & -\vec{p}^T s_0 & 0 \end{pmatrix},$$

and $J = \operatorname{diag}(\mathbb{1}_2, 0, \dots, 0, -\mathbb{1}_2)$. Next we apply reduction [3] with the $\mathbb{Z}_2 \otimes \mathbb{Z}_{2r-4}$, The Cartan involution splits the system of positive roots Δ_+ of $so(2r+1)$ into

$$\Delta_+ = \Delta_+^0 \cup \Delta_+^1, \quad \begin{array}{l} \Delta_+^0 \equiv \{e_1 \pm e_2, \quad e_j - e_k, \quad 3 \leq j < k \leq r\}, \\ \Delta_+^1 \equiv \{e_a - e_j, e_a + e_j, \quad a = 1, 2, \quad j = 3, \dots, r\}. \end{array}$$

The $\mathbb{Z}_2 \times \mathbb{Z}_{2r-4}$-reductions is realized through the automorphism $S_{e_1-e_2}S_{e_3-e_4} \cdot S_{e_{r-1}-e_r}S_{e_r}$. Then Δ_1^+ splits into two orbits for any r which results in a 2-NLSE.

We outline the the explicit form of these 2-NLSE and their Hamiltonian structures. We also formulate the generalized Fourier transforms which linearize them.

References

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