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Phase-plane Analysis of a Model for Cross-feeding in a Chemostat

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Cross-feeding is a symbiotic relationship between microorganisms where one organism consumes metabolites produced by another. A recent mathematical model (Gudelj et al., 2016) for cross-feeding in a chemostat has been developed and parametrised according to experimental data on competing strains of *Escherichia coli* in a laboratory chemostat (Rosenzweig et al., 1994). The strains metabolise glucose in two stages: 1. glycolysis and 2. tricarboxylic acid (TCA) cycle. In step 1, the primary resource, glucose, is partially oxidized and forms an intermediate metabolite, and excess quantities of it are secreted into the environment, and can next serve as secondary energy resource in step 2.

Both strains are able to grow on glucose and on the intermediate metabolite, but the first strain specializes on glucose, and the second strain on the intermediate metabolite (acetate). The metabolite uptaken by the *E. coli* imposes a cost to cell growth. The model is parametrised to model the short-term dynamics of the competing strains, and predicts the existence of stable equlibria where the strains cross-feed for sufficiently large glucose concentrations. The ODE model for the chemostat dynamics is also of mathematical interest due to terms modelling a jump-like transport of the intermediate metabolite through the cell membrane.

To better understand the bifurcation structure of the model and the appearance of cross-feeding equilibria, a bifurcation analysis is made for two specific instances of the model, where the system of 6 ODEs is reduced to a system of 2 ODEs, and permits phase-plane analysis. These simpler cases retain to an extent the complex multistationarity of the full model, and can be used to predict the behaviour of the full model. Furthermore, conditions on model parameters for the nonexistence of cross-feeding are derived analytically.

References

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