Experiences in Solving Discrete PDE-constrained Optimization Problems

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Many inverse and parameter estimation problems can be formulated as PDEconstrained optimization problems, where the constraint is determined by scalar or vector partial differential equations. When discretized, such problems lead to large scale linear or nonlinear algebraic systems and require fast, efficient and robust iterative solution methods.

In recent years much research effort has been devoted to constructing efficient preconditioners for solving algebraic systems arising from various constrained optimization problems. The matrices in the so-obtained linear systems of equations possess a specific block structure and, depending on the target problem, can be definite or indefinite. Various authors have worked on how to precondition these matrices, applying the existing knowledge and the experience of general blockpreconditioners for definite and indefinite systems, arising from the discretization of scalar or vector (systems of) PDEs.

In this talk, examples of PDE-constrained optimization problems with be shown, with various types of control. We briefly survey some well-established and novel preconditioning strategies, illustrated for various classes of PDE-constrained optimization problems, e.g., when the PDE-constraint is given by the Poisson equation, by a convection-diffusion equation, by the stationary Stokes equations and more.

We present some comparative results of the performance of the corresponding preconditioned solvers, both in terms of iterations and in terms of computing time. For the discretization in space (the Finite Element method), the solution methods and the preconditioners are implemented using available toolboxes from open source scientific libraries - deal.ii and Tlilinos.

In this framework, very often the arising algebraic problems are nonlinear. For these cases, an idea how to diminish the computational effort in the nonlinear case will be given too. Difficulties and problems to be addressed in future research will also be mentioned.

This is a joint work with Owe Axelsson, Shiraz Farouq, Anders Sjöberg and Zhao-Zheng Liang.