

## Numerical Simulation of 2D Sine-Gordon Waves

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Systems of perturbed 2D Sine-Gordon equations

$$S(\varphi_{tt} + \alpha\varphi_t + \sin \varphi - \gamma) = \Delta\varphi, \quad (x, y) \in \Omega \subset \mathbb{R}^2,$$

under Neumann boundary conditions and appropriate initial conditions are investigated.  $\Omega$  is a given rectangular domain in  $\mathbb{R}^2$  and  $S$  is a cyclic tridiagonal matrix. Such problems are well known to be very good models for the phase dynamics in inductively coupled intrinsic Josephson junctions [2].

Reliable and efficient second order finite difference schemes are worked out to solve the problems numerically. New type of standing wave solutions, which consist of two parts: static part and oscillating one, are found. Fast Fourier transform analysis shows that the oscillating part is a sum of cavity (linear) and background (nonlinear) oscillation. This new result is in contrast to the results in some previous works (see the review [1]), where the entire nonlinearity is put in the static part of the solution and the analytical computations are made under this assumption.

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## References

- [1] Hu Xiao, and Lin Shizeng. TOPICAL REVIEW: Phase dynamics in a stack of inductively coupled intrinsic Josephson junctions and terahertz electromagnetic radiation. *Supercond. Sci. Technol.*, Vol. 23, 2010, p.053001.
- [2] Sakai S., Bodin P. and Pedersen N. F. Fluxons in thin-film superconductor-insulator superlattices. *Journal of Applied Physics*, Vol. 73, Issue 5, 1993, p.2411–2418.