Invariant Domain Approximations of Nonlinear Hyperbolic Systems

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We propose a technique for approximating nonlinear conservation equations that is based on continuous finite elements. The method is explicit in time, uses unstructured continuous finite elements in space and works in any space dimension. The approximation is maximum principle preserving for scalar equations and invariant domain preserving for systems. The formal accuracy of the method could be improved up to second order and still keep the method maximum principle preserving for scalar equations and positivity preserving for systems with such a property. One novelty of this work is that instead of limiting fluxes and slopes, the stability and the accuracy are achieved by adapting the artificial viscosity.

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