Second International Conference "Mathematics Days in Sofia"
July 10–14, 2017, Sofia, Bulgaria

Subordination Approach to Time-fractional Diffusion-wave Equation of Distributed Order

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Keywords: fractional derivative, distributed-order fractional derivative, diffusion-wave equation, Bernstein function, probability density

Distributed-order diffusion-wave equation is re-examined and new results are presented. We consider continuous as well as discrete distribution of fractional time-derivatives of order between 1 and 2. Equations of this kind appear in the modeling of unidirectional flows of viscoelastic fluids, string vibration with fractional friction, heat conduction with the distributed-order Cattaneo type constitutive law, etc. First, the Cauchy and signaling problems for the spatially one-dimensional equation are studied. Positivity of their fundamental solutions and propagation speed of a disturbance are discussed. Next, we consider the equation with a general linear multi-dimensional spatial differential operator defined in appropriate Banach space and suppose it generates a cosine family. A subordination relation for the considered class of problems is established. This is an integral representation of the solution in terms of the corresponding cosine family and a probability density function, related to the fundamental solution of the equation in one space dimension. The established subordination representation is applied for obtaining regularity results and for the numerical computation of the solution in some particular cases. The analytical findings are supported by numerical work.

Acknowledgements. This work is partially supported by Bulgarian National Science Fund (Grant DFNI-I02/9); and performed in the frames of the bilateral research project between Bulgarian and Serbian Academies of Sciences, "Analytical and numerical methods for differential and integral equations and mathematical models of arbitrary ... order".