

## Fractional Derivative Model for Diffusion-controlled Adsorption at Liquid/Liquid Interface

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In the models of surfactant transport in multiphase flows the reaction-advection-diffusion equations in the continuous phases are coupled via adsorption on the interface. In this study the existing models are generalized in two directions. First, the model of a gas/liquid surface is extended to one in the case of a liquid/liquid interface, where two liquids are in contact. In the second generalization time-fractional models for the description of the diffusion as well as the adsorption are considered. The resulting nonlinear fractional initial-boundary value problem is solved by means of Laplace transform to obtain a nonlinear Volterra-type integral equation for the change of adsorption with time. This is a generalization of the classical integral equation of Ward and Tordai [1]. Numerical techniques for the solution of the integral equation as well as for the original initial-boundary value problem for the diffusion and adsorption equations are discussed.

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## References

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