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Preconditioning Methods for Eddy Current Optimally Controlled Time-harmonic Electromagnetic Problems

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To solve time dependent elliptic problems one normally uses time-stepping methods which, however, are time-consuming and costly. This is because, if an explicit method is used, for reasons of numerical stability one must use very small time steps and if a stable implicit method is used, one must still use somewhat small time-steps in order to get a sufficiently small discretization error. At each time step one must then solve a time-discretized elliptic problem.

As has been observed by many researchers, for time-harmonic problems there is an alternative. One can namely approximate the solution with a truncated Fourier series expansion. Then, for linear problems, the different terms separate and one can solve the given problems for each frequency fully in parallel, so the solution cost measured in elapsed time will essentially be reduced to being proportional to the solution cost of a single elliptic problem.

As has been shown in a number of papers e.g. by Kolmbauer and Langer, such an approach is applicable also for time-harmonic optimal control eddy current electromagnetic problems. For such problems, the discretized problem takes a two-by-two or four-by-four block matrix structure.

The problems are normally three space dimensional, and hence of a very large scale, so iterative solution methods must be used. Then the choice of preconditioning is crucial. For the above problem, we present two types of preconditioners, one based on a block diagonal matrix and one based on the use of preconditioners of two-by-two block form, constructed by adding the off-diagonal block matrices to the primal diagonal block. This method is based on the square matrix property of all subblocks and has been used earlier for optimal control problems and various types of state differential equations, by Axelsson and Neytcheva.

Using a simplified analysis, it is shown that both methods lead to real valued eigenvalues in a very tight interval. This holds uniformly with respect to mesh, method and problem parameters with the exception that the rate of convergence factor of the block diagonal method deteriorates to unity when the problem parameter, the reluctivity, decreases to zero.

Both methods require solution of an elliptic problem on the innermost level, where some standard method can be used. For this some method in available software packages can be used. The two-by-two block matrix preconditioner requires use of coupled inner – outer iterations and for the four-by-four block system, a double outer-inner iteration method is used.

An extensive set of numerical tests are presented. It is shown that there are very few, typically single digits number of outer iterations and also few inner iterations that hold remarkably well for all parameters. This includes problems with both distributed and local controls. For nonlinear problems with sufficiently regular solution a two-grid method can be used.

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