Differential Equation for a Class of Orthogonal Polynomials

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Given numbers $s, n \in \mathbb{N}, n \geq 2$, and the n-th degree monic Chebyshev polynomial of the first kind $\widehat{T}_n(x)$, we consider the polynomial system $\{p_k^{n,s}\}$ "induced" by the modified measure $d\sigma^{n,s}(x) = \widehat{T}_n(x)^{2s} d\sigma(x)$, where $d\sigma(x) = 1/\sqrt{1-x^2} dx$ is the Chebyshev measure of the first kind.

In the first part of this work we determine the coefficients of the three-term recurrence relation for the polynomials $p_k^{2,s}(x)$, but in a unique analytic form which has been proved as valid in [2].

In addition we give differential equation for these class of orthogonal polynomials. Assuming a logarithmic potential, we also give an electrostatic interpretation of the zeros of $p_k^{n,s}(x)$.

In order to verify all complicated formulas, the authors used symbolic computations in MATHEMATICA, with the software package OrthogonalPolynomials, described in [1].

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References

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