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Perturbation Theory for Functions of Self-adjoint Operators

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In the framework of operator theory, estimates of the type

$$\|f(A) - f(B)\| \leq C\|f\|_X\|A - B\| \qquad (*)$$

have been extensively studied. Here A and B are self-adjoint operators in a Hilbert space, f is a function on $\mathbb{R}$, and X is an appropriate function class (the "optimal" class X is still not fully understood). Estimates of this type require a certain degree of smoothness of f: the class X must include $C^1(\mathbb{R})$. They are closely related to the theory of the spectral shift function for the pair of operators A, B.

I will give a brief survey of what is known about $f(A) - f(B)$ when f fails to be smooth; the key example will be a function f with a jump discontinuity. Even though it is probably not possible to say anything interesting about this for general operators A and B, one can make much progress in the framework of scattering theory, when A and B satisfy some standard assumptions of Kato smoothness type. A typical example of this is the free and the perturbed Schrödinger operators $A = -\Delta$, $B = -\Delta + V$ in $L^2(\mathbb{R}^d)$, $d \geq 1$, where the potential $V = V(x)$ decays sufficiently fast at infinity.

In this framework, one can (i) improve the estimate $(*)$ by reducing the smoothness assumptions on f; (ii) describe in detail the spectrum of $f(A) - f(B)$ when f has a jump discontinuity. This description involves the spectrum of the scattering matrix for the pair A, B.

Some of the results I will mention are joint work with Dmitri Yafaev (University of Rennes 1) and others are joint work with Rupert Frank (Caltech).