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Log-Convexity of Weighted Area Integral Means of H^p Functions on the Upper Half-plane

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In the present work weighted area integral means

$$M_{p,\varphi}(f;t) = \frac{\int_1^t \varphi'(y) \int_{-\infty}^{\infty} |f(x+iy)|^p dxdy}{\int_1^t \varphi'(y)dy}$$

are studied and it is proved that the function $t \to \log M_{p,\varphi}(f;t)$ is convex in the case when f belongs to a Hardy space on the upper half-plane and the derivative $\varphi'(t)$ of the function φ equals either t^{-a} or e^{-at}, where $t > 0$, $a > 0$.

Weighted area integral means $M_{p,\varphi}$ are studied in a series of papers by K. Zhu, Ch. Wang, J. Xiao [1–5]. In their papers the following two cases are studied either f is a holomorphic function on the unit disk and φ' is $(1-|z|^2)^{-a}$ or f is a holomorphic function on the whole complex plane and φ' is $e^{-a|z|^2}$.

Now, in this work, their method, complemented with some minor modifications, is applied in the new case when f is a holomorphic function on the upper half-plane.

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