Log-Convexity of Weighted Area Integral Means of $H^p$ Functions on the Upper Half-plane

Martin Stanev

Department of Mathematics and Physics, University of Forestry
Sofia, Bulgaria
martin_stanev@yahoo.com

Keywords: log-convexity, weighted area integral means, holomorphic function, upper half-plane

In the present work weighted area integral means

$$M_{p,\varphi}(f; t) = \frac{\int_1^t \varphi'(y) \int_{-\infty}^{\infty} |f(x + iy)|^p dx dy}{\int_1^t \varphi'(y) dy}$$

are studied and it is proved that the function $t \to \log M_{p,\varphi}(f; t)$ is convex in the case when $f$ belongs to a Hardy space on the upper half-plane and the derivative $\varphi'(t)$ of the function $\varphi$ equals either $t^{-a}$ or $e^{-at}$, where $t > 0, a > 0$.

Weighted area integral means $M_{p,\varphi}$ are studied in a series of papers by K. Zhu, Ch. Wang, J. Xiao [1–5]. In their papers the following two cases are studied either $f$ is a holomorphic function on the unit disk and $\varphi'$ is $(1 - |z|^2)^{-a}$ or $f$ is a holomorphic function on the whole complex plane and $\varphi'$ is $e^{-a|z|^2}$.

Now, in this work, their method, complemented with some minor modifications, is applied in the new case when $f$ is a holomorphic function on the upper half-plane.

References


