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The Calabi-Yau Problem on the Kodaira-Thurston Manifold

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The Calabi-Yau equation is a PDEs system whose study goes back to the celebrated Calabi conjecture proved by Yau in 1978. Recently, Donaldson has described how the equation could be generalized in a natural way to the setting of 2-forms on 4-manifolds. Donaldson's program, if carried out, would lead to many new and important results in symplectic geometry. Given a 4-dimensional compact symplectic manifold (M, Ω) together an $\Omega-$compatible almost-complex structure J, the Calabi-Yau equation consists in

$$(\Omega + d\alpha)^2 = \mathrm{e}^F\,\Omega^2, \quad Jd\alpha = d\alpha$$

where $F \in C^\infty(M)$ is given and α is a unknown 1-form. In contrast to the Kähler case, it is not known if the equation in the almost-complex setting has always a solution. Important results about this problem have been obtained by Tosatti, Weinkove and Yau.

The talk focuses on the study of the Calabi-Yau equation in the Kodaira-Thurston manifold $M = \mathrm{Nil}^3 \times S^1$, viewed as an S^1-bundle over a 3-dimensional torus, when F is invariant by the action of the fiber. It will be showed that in this case the equation reduces to a generalized Monge-Ampère equation on the basis having always a solution.

References

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