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Baire Spaces and Compactness

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By a space we understand a Hausdorff topological space. A subset Z of a space X is τ-compact, if Z is a union of τ compact subsets of X. A subset B of a space X is bounded if for any locally finite family γ of open subsets the set $\{U \in \gamma : U \cap B \neq \emptyset\}$ is finite. A space X is called feebly compact if X is a bounded subset of X. A Baire space is a topological space such that every intersection of a countable collection of open dense sets in the space is also dense. It is well known that every Čech-complete space is a Baire space.

It is true the following general fact:

Theorem 1. *Let Y be a bounded W_δ-subset of a regular space X. Then Y is a Baire space.*

Every G_δ-subset is a W_δ-subset. The W_δ-subsets of compact spaces are characterized as open continuous images of paracompact Čech-complete spaces. There exists a paracompact G_δ-subset of a pseudocompact space which is not a Čech-complete space.

Theorem 2. *A paracompact p-space X is a W_δ-subset of some regular feebly compact space if and only if X is Čech-complete.*

Our main interest is the following question posed by W. Roelcke in 1972: Is there a Hausdorff ω-bounded space which is not Baire? In 1972, Z. Frolik constructed an example of a meager countably compact space and, in 1996, J. R. Porter constructed an example of a countably compact, separable and meager space.

A space X is called:

– $\omega(\tau)$-bounded, if the closure clL in X of every subset $L \subseteq X$ of cardinality $|L| \leq \tau$ is compact;

– a $\sigma_\tau cc^*$-space, if the closure of every τ-compact subset is compact.

Theorem 3. *For each infinite cardinal* τ *there exists a meager Hausdorff* $\sigma_\tau cc^*$*-space* $B(\tau)$ *for which* $d(B(\tau) = \tau^+$.

However, the following problems remain unsolved:

Question 1. *Is there a countably compact or an* ω*-bounded* k*-space which is not Baire?*

Question 2 (W. Roelcke). *Is there a sequentially compact space which is not Baire? Is there a sequentially compact* ω*-bounded space which is not Baire?*