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Quasi-Einstein Manifolds

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A quadruple (M, g, f, μ), where (M, g) is a pseudo-Riemannian manifold of dimension n, f is a smooth function on M, and $\mu \in \mathbb{R}$, is said to be a *generalized quasi-Einstein manifold* if the tensor $\mathrm{Hes}_f + \rho - \mu df \otimes df$ is a multiple of the metric, i.e.

$$\mathrm{Hes}_f + \rho - \mu df \otimes df = \lambda\, g \quad \text{for some} \quad \lambda \in \mathcal{C}^\infty(M)\,. \tag{1}$$

There are several important families of generalized quasi-Einstein manifolds previously investigated in the literature: Einstein metrics, gradient Ricci almost solitons, conformally Einstein metrics, static spacetimes, (see also [1]).

Equation (1) provides information on the curvature of the manifold since it involves the associated Ricci tensor. We shall impose various conditions on the Weyl tensor to obtain related families of generalized quasi-Einstein manifolds. One could assume, for example, that (M, g) is locally conformally flat; this condition turns out to be quite restrictive [4]. Other natural conditions on the conformal curvature were previously considered for 4-dimensional manifolds, namely half conformal flatness (i.e., (M, g) is either self-dual or anti-self-dual). One has that half conformally flat quasi-Einstein manifolds are locally conformally flat in the Riemannian setting [3, 5]. The key point in this analysis is that, in definite signature, the level hypersurfaces of the potential function are non-degenerate and have constant sectional curvature. However, this need no longer hold true if the signature is indefinite.

In this work, we shall examine 4-dimensional generalized quasi-Einstein manifolds in neutral signature $(2, 2)$ aimed to describe the local structure of those which are half conformally flat, but not locally conformally flat.

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References

[1] M. Brozos-Vázquez and E. García-Río, Indiana Univ. Math. J., 65 (2016), 1921–1943.

[2] M. Brozos-Vázquez, E. García-Río, P. Gilkey and X. Valle-Regueiro, Half conformally flat generalized quasi-Einstein manifolds, arXiv:1702.06714 [math.DG].

[3] G. Catino, A note on four-dimensional (anti-)self-dual quasi-Einstein manifolds, *Differential Geom. Appl.*, 30 (2012), 660–664.

[4] G. Catino, C. Mantegazza, L. Mazzieri and M. Rimoldi, Locally conformally flat quasi-Einstein manifolds, *J. Reine Angew. Math.*, 675 (2013), 181–189.

[5] Y. Deng, A note on generalized quasi-Einstein manifolds, *Math. Nachr.*, 288 (2015), 1122–1126.