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Dirichlet Sets and Arbault Sets of the Circle Group

Giuseppina Barbieri, Dikran Dikranjan,
Anna Giordano Bruno, Hans Weber

Department of Mathematics, Udine University
Udine, Italy
pinuccia.barbieri@gmail.com, dikran.dikranjan@uniud.it,
anna.giordanobruno@uniud.it, hans.weber@uniud.it

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A subset A of the circle group $\mathbb{T}$ is a *Dirichlet set* if there exists an increasing sequence $\mathbf{u} = (u_n)_{n\in\mathbb{N}_0}$ in $\mathbb{N}$ such that $\|u_n x\|$ uniformly converges to 0 on A. The subgroup $t_{\mathbf{u}}(\mathbb{T}) := \{x \in \mathbb{T} : \|u_n x\| \to 0\}$ is called a *characterized subgroup* of $\mathbb{T}$, while subsets of $t_{\mathbf{u}}(\mathbb{T})$ are called *Arbault sets*. The interest in these sets stems from descriptive set theory, number theory, harmonic analysis and topology [1,2].

Using strictly increasing sequences $\mathbf{u}$ in $\mathbb{N}$ such that u_n divides u_{n+1} for every $n \in \mathbb{N}$, we find in $\mathbb{T}$ a family of closed perfect D-sets that are also Cantor-like sets. Moreover, we write $\mathbb{T}$ as the sum of two closed perfect D-sets. As a consequence, we solve an open problem by showing that $\mathbb{T}$ can be written as the sum of two of its proper characterized subgroups, i.e., $\mathbb{T}$ is factorizable (see [3] for a related result). Moreover, we describe all countable subgroups of $\mathbb{T}$ that are factorizable and we find a large class of uncountable characterized subgroups of $\mathbb{T}$ that are factorizable.

References

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