Almost Contact B-metric Structures and the Bianchi Classification of the Three-dimensional Lie Algebras

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The present work deals with differential geometry of almost contact B-metric manifolds. These manifolds are the odd-dimensional extension of almost complex manifolds with Norden metric and a neutral-signature analogue of almost contact manifolds with compatible metric. An object of special interest is the case of the lowest dimension of the considered manifolds – dimension 3.

It is adduced the well-known Bianchi classification of the three-dimensional real Lie algebras. We consider the Lie groups which corresponds to the Lie algebras of each of the Bianchi classes. These Lie groups we equip with an almost contact B-metric structure and we establish the belonging of each of the constructed almost contact B-metric manifolds to the respective class of the Ganchev-Mihova-Gribachev classification of the considered manifolds. In this way, it is considered a relation between the classes in the Bianchi classification of three-dimensional real Lie algebras and the classes of a classification of the almost contact B-metric manifolds.

The present work gives some geometrical characteristics of considered manifolds in certain special classes.