

Canonical Weierstrass Representations for Minimal Surfaces in Four-dimensional Euclidean or Minkowski Space

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We realize the following scheme of investigation of minimal surfaces in four-dimensional Euclidean or Minkowski space:

Any minimal surface of general type locally admits geometrically determined parameters – canonical parameters. In such parameters the Gauss curvature and the normal curvature satisfy a system of two natural partial differential equations and determine the surface up to a motion. For any minimal surface parameterized by canonical parameters we obtain Weierstrass representations – canonical Weierstrass representations. These Weierstrass formulas allow us to solve explicitly the system of natural partial differential equations and to establish geometric correspondence between minimal surfaces of general type, the solutions to the system of natural equations and pairs of holomorphic functions in the Gauss plane.

On the base of these correspondences we obtain that any minimal surface of general type in Euclidean 4-space determines locally a pair of two minimal surfaces in Euclidean 3-space and vice versa.

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