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Lorentz Surfaces with Parallel Normalized Mean Curvature Vector Field in Pseudo-Euclidean 4-Space with Neutral Metric

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A basic class of surfaces in Riemannian and pseudo-Riemannian geometry are the surfaces with parallel mean curvature vector field, since they are critical points of some natural functionals and play important role in differential geometry, the theory of harmonic maps, as well as in physics. A natural extension of the class of surfaces with parallel mean curvature vector field are surfaces with parallel normalized mean curvature vector field. A surface M in a Riemannian or pseudo-Riemanian manifold is said to have parallel normalized mean curvature vector field if the mean curvature vector H is non-zero and the unit vector in the direction of the mean curvature vector is parallel in the normal bundle.

We study Lorentz surfaces in the pseudo-Euclidean 4-space with neutral metric $\mathbb{E}^4_2$. On any Lorentz surface with parallel normalized mean curvature vector field we introduce special geometric (canonical) parameters that allow us to prove the fundamental existence and uniqueness theorem for this class of surfaces in terms of three invariant functions.

Our main result states that any Lorentz surface with parallel normalized mean curvature vector field is determined up to a rigid motion in $\mathbb{E}^4_2$ by three invariant functions $\lambda(u,v)$, $\mu(u,v)$, $\nu(u,v)$ satisfying a system of three natural partial differential equations:

$$\begin{aligned}
&\nu_u = -\lambda_v + \lambda(\ln|\mu|)_v;\\
&\nu_v = \lambda_u - \lambda(\ln|\mu|)_u;\\
&|\mu|\Delta^h \ln|\mu| = 2\varepsilon(\lambda^2 - \mu^2 + \nu^2),
\end{aligned}$$

where $\varepsilon = \pm 1$. This theorem solves the Lund-Regge problem for the class of surfaces with parallel normalized mean curvature vector field in $\mathbb{E}^4_2$.

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References

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