Second International Conference "Mathematics Days in Sofia"
July 10–14, 2017, Sofia, Bulgaria

Some Applications of Classical Invariant Theory to Combinatorics

Leonid Bedratyuk

Khmelnytskyi National University, Ukraine
leonid.uk@gmail.com

Keywords: classical invariant theory, locally nilpotent derivations, special functions, polynomial identities, graph invariants

MSC 2010: 13A50, 11B68, 05A19, 05C30

We consider two application of classical invariant theory to combinatorial problems.

Let $a_{n,i}$ be the number of simple graphs with n vertices and k edges. Let

$$g_n(z) = \sum_{i=0}^{m} a_{n,\,i} z^i, m = \binom{n}{2},$$

be the ordinary generating function for the sequence $\{a_{n,i}\}$. Denote by $[n]^{(2)}$ the set of 2-subsets of $[n]$. Let S_n be the permutation group on the set $[n]$. The pair group of S_n, denoted $S_n^{(2)}$ is the permutation group induced by S_n which acts on $[n]^{(2)}$. We offer the following formula for the generating function $g_n(z)$:

$$g_n(z) = \frac{1}{n!} \sum_{\alpha \in S_n^{(2)}} \frac{\det(\mathbf{1}_m - \alpha \cdot z^2)}{\det(\mathbf{1}_m - \alpha \cdot z)}.$$

Let $\{P_n(x)\}, n = \deg P_n(x)$, be a system of polynomials over $\mathbb{Q}$. We are interested in finding polynomial identities for the system of polynomials, i.e., identities of the form

$$F(P_0(x), P_1(x), \ldots, P_n(x)) = 0,$$

where F is some polynomial in $n+1$ variables. Using methods of classical invariant theory a general approach to find identities for some well-known families of polynomials (Bernoulli, Euler, Hermite, Fibonacci, Lucas, Kravchuk polynomials) is proposed.