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Calculation of Fundamental Unit for an Especial Type of Real Quadratic Number Fields

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In Number Theory, the notion of the quadratic fields is difficult task. There are many different approaches such as genus theory, composition of binary quadratic forms, and class field theory as a developmental tool for quadratic fields. Moreover, many books and papers on the number theory apply many different methods like continued fraction expansions, class number, regulators in the class group, etc... Recently, class number which is very difficult to calculate is used in the cryptology and security.

In this paper, we determine the real quadratic fields coincide with positive square free integers d including specific continued fraction expansion of integral basis element in the case of $d \equiv 2, 3 \pmod 4$ or $d \equiv 1 \pmod 4$, where $\ell(d)$ is the period length of continued fraction expansion. Besides, we deal with determining the fundamental unit and Yokoi's d-invariants n_d and m_d in the relation to continued fraction expansion of w_d, we also give several numerical tables to support our results.

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