Second International Conference "Mathematics Days in Sofia"
July 10–14, 2017, Sofia, Bulgaria

# On the Covering Number of Small Symmetric, and Alternating Groups, and Some Sporadic Simple Groups

**Daniela Nikolova-Popova[1], Luise-Charlotte Kappe[2], Eric Swartz[3], Spyros Magliveras[1], Michael Epstein[1]**

[1]*Department of Mathematical Sciences, Florida Atlantic University, Boca Raton, FL 33431, USA*
dpopova@fau.edu, spyros@fau.edu, mepstein2012@fau.edu

[2]*Department of Mathematical Sciences, State University of New York at Binghamton, Binghamton,NY 13902-6000, USA*
menger@math.binghamton.edu

[3]*Department of Mathematics, College of William and Mary, Williamsburg, VA 23187, USA*
easwartz@wm.edu

**MSC 2010:** 20D06, 20D60, 20D08, 20-04; secondary 20F99.

We say that a group $G$has a finite covering if $G$is a set theoretical union of finitely many proper subgroups. The minimal number of subgroups needed for such a covering is called the covering number of $G$denoted by $\sigma(G)$:

Let $S_n$ be the symmetric group on $n$letters. For odd $n$Maroti determined $\sigma(S_n)$ with the exception of $n = 9$, and gave estimates for $n$ even showing that $\sigma(S_n) \leq 2n - 2$. We show that $\sigma(S_8) = 64$, $\sigma(S_{10}) = 221$, $\sigma(S_{12}) = 761$. We also show that Maroti's result for odd $n$ holds without exception proving that $\sigma(S_9) = 256$. We establish in addition that the Mathieu group $M_{12}$ has covering number 208, and improve the estimate for the Janko group $J_1$ given by P. E. Holmes.

In another paper, we establish the covering number of $A_9$, and $A_{11}$. As of now, the smallest values of $n$ for which the covering numbers of $S_n$, and $A_n$ are not known are $n = 14$, and $n = 12$, respectively.

The methods we use involve *GAP* calculations, incidence matrices and linear programming.

The coverings turn out to be dependent on the arithmetic nature of $n$. Some results for larger classes of Sn have been established.

## References


[1] Luise-Charlotte Kappe, Daniela Nikolova-Popova and Eric Swartz, On the covering number of small symmetric groups and some sporadic simple groups, *Groups, Groups Complex. Cryptol.* 2016; 8 (2):135–154.

[2] M. Epstein, S. Magliveras and D. Nikolova, The covering numbers of $A_9$ and $A_{11}$, *J. Combin. Math. Combin. Comput.*, to appear in May, 2017.

[3] E. Swartz, On the covering number of symmetric groups having degree divisible by six, *Discrete Math.*, 339(11), 2016, 2593–2604.

[4] The GAP Group, GAP-Groups, Algorithms, and Programming, Version 4.4.7, 2006, `http://www.gap-system.org`.

[5] Gurobi Optimizer Reference Manual, Gurobi Optimization, Inc., 2014, `http://www.gurobi.com`.