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Decomposition of Cohomology of Vector Bundles on Homogeneous Ind-spaces

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A *locally semisimple ind-group* G over $\mathbb{C}$ is the inductive limit $G = \varinjlim G_n$ of embeddings of connected semisimple algebraic groups

$$G_1 \subset G_2 \subset \cdots \subset G_n \subset \cdots$$

Let G be a locally semisimple ind-group and P be a parabolic subgroup. We introduce a class of finite-dimensional P-modules called *strongly finite P-modules.* Then, for any strongly finite P-module E we consider the homogeneous vector bundle $\mathcal{O}_{G/P}(E^*)$ on G/P induced by the dual P-module E^*. We show that the nonzero cohomology groups of $\mathcal{O}_{G/P}(E^*)$ decompose as direct sums of cohomologies of bundles of the form $\mathcal{O}_{G/P}(R)$ for (some) simple constituents R of E^*. In the classical case of semisimple algebraic groups G, this result is a consequence of the Bott-Borel-Weil theorem and Weyl's semisimplicity theorem. In the case of locally semisimple ind-groups G we consider, there is no relevant semisimplicity theorem. Instead, we show that the cohomologies of the bundles $\mathcal{O}_{G/P}(R)$ are injective objects in a certain category of $\mathfrak{g} = \mathrm{Lie}(G)$-modules. In the end, for the class of diagonal locally semisimple ind-groups G we give explicit criteria and examples for a P-module E to be strongly finite. The talk is based on a joint work with Ivan Penkov.

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