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On the Brauer p-dimension of a Henselian Discrete Valued Field of Prime Residual Characteristic p

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Let E be a field, $s(E)$ the class of associative finite-dimensional central simple E-algebras, and for each $A \in s(E)$, let $[A]$ be the equivalence class of A in the Brauer group $\mathrm{Br}(K)$. The exponent (or period) $\exp(A)$, i.e. the order of $[A]$ in $\mathrm{Br}(E)$, and the Schur index $\mathrm{ind}(A)$ are basic invariants both of A and $[A]$. It is known that set of Brauer p-dimensions $\mathrm{Brd}_p(E)$, $p \in \mathbb{P}$, where $\mathbb{P}$ is the set of prime numbers, contains an essential (frequently, complete) information on the pairs $\mathrm{ind}(A)$, $\exp(A)\colon A \in s(E)$. The absolute Brauer p-dimension $\mathrm{abrd}_p(E)$, i.e. the supremum of $\mathrm{Brd}_p(R)$, where R ranges over the class of finite separable extensions of E, has influence on the set of index-exponent p-primary F-pairs, for every finitely-generated transcendental field extension F/E. For example, $(p^{\mu'}, p^{\mu})$ is such a pair, provided that $\mathrm{abrd}_p(E)$ is infinity and μ', μ are integers with $\mu' \geq \mu > 0$ [3], Theorem 2.1.

In this talk we focus our attention on $\mathrm{Brd}_p(E)$, assuming that E has a Henselian discrete valuation v with a residue field $\widehat{E}$ of characteristic $p > 0$. We first show that $\mathrm{Brd}_p(E) = \infty$ if and only if $\widehat{E}/\widehat{E}^p$ is an infinite extension, $\widehat{E}^p$ being the subfield of p-th powers of elements of $\widehat{E}$. Henceforth, we consider the case where $[\widehat{E}\colon \widehat{E}^p] = p^n$, for some integer $n \geq 0$. Our main results agree with the Bhaskhar-Haase Conjecture (abbr, (BHC)) [1], which states that $n \leq \mathrm{Brd}_p(E) \leq n+1$. This has been proved in [1], for $n \leq 3$, and in general, one can show $\mathrm{Brd}_p(E) \leq 2n$ [5]. Our main result proves (BHC) in the case where char$(E) = p$ or $\widehat{E}$ is an n-dimensional local field (in the former case, the upper bound on $\mathrm{Brd}_p(E)$ is an easy consequence of a well-known theory due to Albert, and in the latter one, this bound has been found in [4]). When char$(E) = 0$, the main result of the talk states that $\mathrm{abrd}_p(E) \geq n$ and $\mathrm{Brd}_p(E) \geq n - [n/3] - 1$; in addition, it yields $\mathrm{Brd}_p(E) \geq n - [n/3]$, if $n \neq 5$ or E contains a primitive p-th root of unity. When $n \neq 1, 2, 3$ or 5, our lower bound on $\mathrm{Brd}_p(E)$ is better than the lower bounds found in [1] and [5]. We also obtain that $\mathrm{Brd}_p(E) \leq 1$ if and only if $n = 1$ and $\widehat{E}$ is a p-quasilocal field, in the sense of [2]; special cases of this result have earlier been obtained by Tignol, Aravire-Jacob, Yamazaki and Zheglov. Yamazaki's result [6], Proposition 2.1 (see also [1], Proposition 4.5), is used for proving the sufficiency part of our assertion.

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