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Statistics for High-frequency Observations of a Stochastic Process

Jean Jacod

Université Pierre et Marie Curie, Paris
jean.jacod@upmc.fr

Consider a stochastic process evolving through time, such as the electric potential of a neuron, or the price of a stock. By necessity the observations are discrete in time, perhaps with a measurement error, and we want to make some kind of inference about the law of the process, based on these discrete observations (for simplicity, in this talk we suppose that there is no measurement error, but it would be possible to take them into considerations). In most practical applications the observations have two distinct features. First, the frequency of observations is extremely high, allowing us to use asymptotic methods by letting the frequency go to infinity. Second, in contrast with the classical statistical setting, one cannot repeat the experiment under the same conditions: two different stocks give rise to two price processes with different laws. So we really observe a sampled version of a single path of the process.

Our aim is to present which kind of statistical inference on the stochastic process is possible in this setting, and a few methods to achieve this. We will start by estimating the so-called "volatility" in finance, that is the diffusion coefficient for a stochastic process which is driven by a Brownian motion (a typical model used in mathematical finance). Then we show how it is possible to determine the (possible) jumps of the observed process and study which of their probabilistic properties can be statistically estimated, which emphasis on the "degree of activity" of those jumps. We will in particular present some methods which have been developed very recently toward this aim.