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Automorphisms of Algebras, Multiple Orthogonal Analogs of Classical Orthogonal Polynomials and Bi-orthogonal Ensembles

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Classical orthogonal polynomial systems of Jacobi, Hermite and Laguerre have the property that the polynomials of each system are eigenfunctions of a second order ordinary differential operator. According to a famous theorem by Bochner they are the only systems with this property. Multiple orthogonal polynomials (MOP) are polynomial systems orthogonal with respect to several measures. They are a subject to intensive research in the last years with a lot of applications to approximation theory, random matrices, number theory, etc.

In the talk we discuss methods for construction of MOP with generalized Bochner's property. This means that the polynomials satisfy both finite term recurrence and are eigenfunctions of a fixed differential operator eventually of higher order. The methods are purely algebraic and are based on automorphisms of non-commutative algebras. Applications of the abstract methods include broad generalizations of the classical orthogonal polynomials, both continuous and discrete. This class has essentially all of the properties of the classical orthogonal polynomials, e.g. they have hypergeometric representations, Rodrigues-like formulas, ladder operators, generating functions, Pearson differential equations for the vector of weights, etc.

The classical orthogonal polynomials have a lot of applications in mathematics, engineering, physics, etc. One of these is in the theory of random matrices.

The newly constructed polynomial systems also have applications to random matrices, some of which are already known. We suggest a unified approach to a class of biorthogonal ensembles, which contains the recently studied by A. Kuijlaars and L. Zhang products of Ginibre matrices, as well as the Borodin-Muttalib ensembles. The most important novel feature is the use of the differential equation, which allows to construct the orthogonality measures of the MOP and describe the correlation kernels.

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