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Wave Asymptotics for Manifolds with Cylindrical Ends

Kiril Datchev

Purdue University, West Lafayette,
Indiana, United States
kdatchev@purdue.edu

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Asymptotics of solutions to the wave equation on a manifold M are well known to be related to the geometry of M and to dynamical properties of its geodesic flow.

If M is a bounded Euclidean domain Ω, or more generally a compact manifold, these solutions can be written as sums of oscillating eigenstates of the Laplacian. The frequencies of oscillation (eigenvalues) are related to the geometry and dynamics by, for example, Weyl asymptotics. Such a setting is a *closed system*, because all geodesics are bounded, and so none escape.

If M is instead the complement of Ω, the spectrum of the Laplacian is continuous rather than discrete, and we attempt to write solutions to the wave equation as sums of oscillating and decaying resonant states, up to a small remainder. In many situations this has been done, with the frequencies of oscillation and rates of decay (resonances) and the size of the remainder depending on geometry and dynamics. Similar results obtain when M is a manifold with suitable asymptotically Euclidean ends, or even some more general ends. Such a setting is an *open system*, because modulo a compact set all geodesics escape to infinity.

An interesting intermediate situation is a manifold with infinite cylindrical ends, which we call a mixed system. In this case the continuous spectrum has increasing multiplicity as energy grows, and in general embedded resonances and eigenvalues can accumulate at infinity. However, we prove that if geodesic trapping is sufficiently mild, then such an accumulation is ruled out, and moreover the cutoff resolvent is uniformly bounded at high energies. We deduce from this the existence of resonance free regions and compute asymptotic expansions for solutions of the wave equation.

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